$$p_{mn}=exp\{-\Sigma_{i=1}^{j}\lambda_i(\xi_m^{\,2}+\eta_n^{\,2})^{\beta i}\}, \text{ for } 0\leqq m,\ n\leqq N-1. \qquad \text{Equation (23)}$$

Then form:

$$G(\xi_m,\ \eta_n)-P^2_{mn}G(\xi_m,\ \eta_n)/\{P^2_{mn}+\omega^2+K^{-2}|1-P^8_{mn}|^2\} \qquad \text{Equation (24)}$$

where $0\leqq m,\ m\leqq N-1$.

In partial restoration block **18** of image restoration method **10**, a partial restoration at t, $0\leqq t\leqq 1$, is constructed by forming:

$$G(\xi_m,\ \eta_n)=p_{mn}^{\,5-1}G(\xi_m,\ \eta_n),$$

where $0\leqq m,\ n\leqq N-1$.

In inverse transform block **20** an inverse two-dimensional fast Fourier transform is performed upon $G(\xi_m,\ \eta_n)$ which was constructed in restoration block **18**. The inverse transform shown in inverse transform block **20** may be obtained by forming

$$f(x_j,\ y_k,\ t)=2^{-2}\Sigma_{m=0}^{N-1}G(\xi_m,\ \eta_n)\exp(\{2\pi i(mj+nk)/N\},$$

where $0\leqq j,\ k\leqq N-1$.

The centering of centering block **12** is then undone in block **22** of image restoration method **10** by forming

$$f(x_j,\ y_k,\ t)=(-1)^{j+k}f(x_j,\ y_k,\ t),$$

where $0\leqq j,\ k\leqq N-1$. Execution of image restoration method **10** may then return to partial restoration block **18** for any other desired value of t.

The result of performing the operations of block **22** is a partial restoration of an image according to image restoration method **10** at the preselected value of t. The user of image restoration method **10** may dispense with partial restoration and proceed directly to full restoration by setting t=0 when restoration block **18** is executed.

The scaling of the fast Fourier transforms in blocks **14**, **20** was chosen so as to correspond to continuous Fourier transform operations in a manner well known to those skilled in the art. However, the factor $\Delta x\Delta y$ may be omitted from the forward transform block **14** provided 1^{-2} is replaced with N^{-2} in reverse transform block **20**. It is straightforward to modify the procedure so as to handle rectangular $N_1\times N_2$ images; see e.g., R. C. Gonzalez and P. Wintz, *Digital Signal Processing,* Second Edition, Addison-Wesley, Reading, Mass. (1987).

The system of method **10** may be implemented with tentative values of some of the parameters λ_i, β_i, ϵ, M, K, s. Image restoration method **10** may be repeated with adjusted values. A sequence of partial restorations as $t\downarrow 0$ is a useful option in that context because noise and ringing usually increase as $t\downarrow 0$. Thus, by performing the restoration in 'slow-motion,' an experienced user may more easily determine the influence of various parameter values and more quickly arrive at corrected values. Tikhonov restoration is obtained by setting s=0 in filter construction block **16**.

To implement Method **10**, it is not necessary to know the optical system's point spread function p(x,y), or its optical transfer function $\hat{p}(\epsilon,\eta)$, in analytical form. Thus, when the imaging system's point spread function p(x,y) is obtained experimentally as a digitized array $p(x_j,y_k)$, one may use the discrete Fourier transform to create the digitized optical transfer array P_{mn}, for use in Equations (23) and (24). In certain cases, method **10** can be implemented even if the system's optical transfer function is not of the form expressed by Equation 5A.

A good starting value for the ratio $\omega=\epsilon/M$ in method **10** can be obtained using "L-curve analysis" as taught in P. C. Hansen, *Analysis of discrete ill-posed problems by means of the L-curve,* SIAM Review, Volume 34, (1992), pp. 561–580.

To locate a good starting value for the parameter K in method **10**, proceed as follows. First fix s at a small value, such as s=0.01 or s=0.001. Next, observe from the form of the filtering function in Equation (24) that with fixed s, the filter approaches the Tikhonov filter if K becomes too large. On the other hand, if K is made too small, the Equation (24) becomes a very low-pass filter which oversmooths the image and destroys information. One can use these observations to visually find the optimal value of K, by performing several restorations each with a different value of K, keeping s and ω fixed. Values of K which reproduce the noisy Tikhonov solution are too large. Values of K which oversmooth the image are too low. A range of K values is quickly found which deblur the image while minimizing noise amplification. The optimal value of K can then be located by further trial restorations in that range of values.

Contour plots of light intensity values are helpful in the above interactive search for K values. Such plots complement the information contained in the usual photographic image displays. In particular, the onset of noise is more easily picked up in the contour plots than it is in the photographic images.

Method **10** can be implemented given the digitized array of a point spread function $p(x_j,y_k)$. We do not need to know the point spread function analytically. We also do not need to know the optical transfer function in analytic form. We can always use fast Fourier transform algorithms to convert the digitized print spread function array into a digitized optical transfer function array for input into method **10**.

The deblurring experiment discussed in Section 4.3 of the attached manuscript *Overcoming Holder Continuity in Ill-Posed Continuation Problems,* illustrates the preceding developments set forth with respect to image restoration method **10**. At the same time, the experiment demonstrates the soundness of the computational implementation. Thus, image restoration method **10** may be reduced to practice as shown, for point spread functions in class G.

It is believed that image restoration method **10** may be advantageously applied in biomedical imaging, night vision systems, undersea imaging, imaging through the atmosphere, remote sensing, high definition television, as well as several other scientific and industrial applications where electron optics and class G point spread functions play a major role. Two key ideas make image restoration method **10** useful as a diagnostic tool in these and other fields. First, the substantial qualitative improvement in the full restoration that results from the additional constraint in Equation (16). Second, the display of the evolution of the restoration as t approaches 0. Together, these factors offer the possibility of greatly improved diagnostic capabilities, and provide a useful addition to current image restoration technology.

It is believed that method **10** is useful in this manner because it is based on mathematical tools which are different from prior art restoration algorithms. In contrast to the input-output linear system theory familiar to researchers with backgrounds in electrical engineering or computer science, and exemplified by Tikhonov restoration or Wiener filtering, method **10** is based upon partial differential equations, semi-group theory, and the mathematics of dif-